Linear Reoriented Coordinates Method

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***Abstract*— This paper presents a non-traditional method and algorithm to calculate the inverse solution for a one-dimensional function without the diffeomorphism property. The proposed method is called the Linear Reoriented Coordinates Method (LRCM). The LRCM is a very powerful and useful too to calculate the symbolic solutions for transcendental functions where the inverse function is not possible to calculate using other traditional methods and only analytic solutions can be calculated but symbolic solutions are not possible to obtain. The description and conditions for the application of the method are presented in the paper. Three of the applications presented in the paper will be to optimize the maximum rectangular area for a floorplan for an 8-bit A/D converter given space constraints, to determine the maximum power for a photovoltaic module (PVM) and for a fuel cell. In both applications, it is not possible to calculate the maximum values using only differential calculus. Finally, examples and simulations for the LRCM are presented.**

I. Introduction

For the last several centuries, the solution for transcendental functions has been a challenge for physics, engineers and mathematicians. A transcendental function is defined as function which does not satisfy a polynomial equation, whose coefficients are polynomials themselves, (i.e. $F(x) = \alpha_n x^n + ... + \alpha_1 x + \alpha_0, \forall \alpha_i \in \Re$). Some examples for transcendental functions are exponential functions, logarithmic functions, and trigonometric functions [1]. The most useful transcendental functions for science are exponential functions. They have an incredible number of applications, but it is not always possible to solve them symbolically. Examples for modeling with transcendental functions are in RLC circuits, fuel cells, photovoltaic modules [2], maximum area for space optimization given shape constraints [3], [4], [5], neural networks [6], robotics [7], etc.

Unfortunately, the only way to solve them it is numerically, sometimes with long and tedious iterations and the use of computers with complex algorithms [7], [8], [9], [10]. Now, for any kind of function, the traditional and effective way to calculate the maximum or minimum values is using differential calculus. But in many cases in physical sciences, engineering or math when it is required modeling using transcendental functions are very complex to work with them.

If a function $y = f(x)$ has the diffeomorphism property then it is possible to obtain the maximum value y_{max}. It is determined when the first derivative of $f(x)$ is calculated with respect to x, then the function $f'(x) = 0$ is solved with respect to x to find the optimal x and y_{max}. Diffeomorphism is defined as a map between manifolds which is differentiable and has differentiable inverse. In other words, for a one-dimensional system, it is a change of coordinates that does not change information given by the original system [11]. A function $f(x)$ has the diffeomorphism property if it is smooth, it has an inverse and the inverse is smooth. If a function has the diffeomorphism property, then it is possible to find the inverse for the given function. The inverse function is defined as follows.

If $f : X \to Y$ is $1-1$ and onto then the correspondence that goes backwards from Y to X is also a function and is called f inverse, denoted f^{-1}. This map is easily described by $f^{-1} : Y \to X$ and $f^{-1}(y) = x$ if and only if $y = f(x)$. This relationship is easy to remember for a real function since switching coordinates of a point in the plane puts us at the reflection of the original point about the line $y = x$. Thus the graph of f^{-1} must be the reflection of the graph of f about the line $y = x$. This is a great help if the graph of f is already known. It's the $1-1$ condition that is really critical for constructing an inverse function. If f is $1-1$ but not onto we can simply replace the codomain with the range $f(X)$ so that $f : X \to f(X)$ in then $1-1$ and onto so we can talk about an inverse $f^{-1} : f(X) \to X$. The domain of the function is equal to the range of the inverse and the range of the function is equal to the domain of the inverse. Finally, a unique inverse only will exist in $1-1$ functions or the unique inverse will exists only over the restricted domain [1].

Unfortunately, it is not always possible to find the symbolic inverse for a given function, $x = f^{-1}(y)$, [12]. But then the question arises, is it at least possible to approximate the inverse of one-dimensional function and how good it is this approximation? To answer these questions, this paper proposes a non-traditional method to approximate the symbolic inverse for one-dimension transcendental functions. Also, the paper provides the different conditions where the method can be applied and which type of functions can be satisfied.

II. Rolle's and Lagrange's Theorems

The main idea for the LRCM (Fig. 1) is based in the Rolle's and Lagrange's Theorems (Mean Value Theorem or Fundamental Theorem Calculus) and it is valid in any domain $[a \quad b]$ but first we need to understand if it is possible to approximate the inverse of a one-dimensional function. The Lagrange Inversion Theorem (LIT) [1] determines the Taylor series expansion of the inverse function of analytic function. Consider the function, $y = f(x)$, where if f is analytic at a point x_0 and $f'(x_0) \neq 0$. Then it is possible to invert or solve the equation for y, $x = f^{-1}(y) = h(y)$ where h is analytic at the point $y_0 = f(x_0)$. The reversion of series is given by the series expansion of $h(y)$ in (1).

$$h(y) = x_0 + \sum_{k=1}^{\infty} \frac{(y-y_0)^k}{k!} \cdot \frac{\partial^{k-1}}{\partial x^{k-1}} \left(\frac{(x-x_0)^k}{(f(x)-y_0)^k} \right) \Bigg|_{x=x_0} \quad (1)$$

This equation will give the inverse function $h(y)$, but unfortunately it is required to do long calculations. Depending the type of functions (or the use of computers), the result most of the time will be an infinite series polynomial (Taylor series). In the case of transcendental functions, it will be required to take into consideration the restrictions on the domain making it difficult to calculate the inverse.

But how can these problems be solved and how can an approximate inverse function be found without the use of Taylor series, long iterations and be a good approximation? The Linear Reoriented Coordinates Method (LRCM) can be a solution for these problems for at least a family of functions!

Theorem 3.1 (Rolle's Theorem, Fig. 2). If $f(x)$ is differentiable on $(a, \ b)$, continuous on $[a, \ b]$ and $f(a) = f(b)$, then $\exists$ c-value in $(a, \ b)$ such that $f'(c) = 0$.

Corollary 3.1 (Modified Rolle's Th.). If for $f(x)$ $\exists!$ maximum value f_{max} then $\exists!$ $x(f'(x_{op}) = 0)$ in $\Re \times [0 \ \ x_{max}]$.

Theorem 3.2 (Lagrange's Theorem, Fig. 3). If g is continuous and differentiable on $[a, \ b]$, then $\exists$ c-value in $[a, \ b]$ such that, $g'(c) = (g(b) - g(a))/(b - a)$.

Corollary 3.2. If $f(x) = x \cdot g(x)$ and $f(x_{op}) = x_{op} \cdot g(x_{op}) = f_{max}$ then $g'(x_{op}) = -g(x_{op})/x_{op}$.

Theorem 3.3 (Cauchy Mean Value Theorem). If g and f are continuous and differentiable on $[a, \ b]$, then c-value in $[a, \ b]$ such that, $f'(c)/g'(c) = (f(b) - f(a))/(g(b) - g(a)))$. The proofs for each theorem and corollary are well known and are skipped in the paper.

III. Linear Reoriented Coordinates Method

A. Description for the LRCM

The LRCM is a method to find the approximate maximum value for a function $f(x)$, where $f'(x) = r(x) = 0$, which cannot be solved using traditional methods of differential calculus, [13]. The LRCM can also be seen as a method to find the approximate symbolic solution x for the equation $r(x) = 0$ without symbolic solutions. The function $f(x)$ is defined as $f(x) = x \cdot g(x)$ and the maximum value of $f(x)$ is defined as f_{max} where $f_{max} = x_{op} \cdot g(x_{op})$ and x_{op} is the optimal value for f_{max}. The main idea for the LRCM is to find the optimal points to calculate f_{max}. These points are $(x_{op}, \ g(x_{op}))$ and are calculated using $g'(x)$ and the linear slope ml of $g(x)$ evaluated at the point x_{op}.

B. Conditions for the LRCM

The necessary conditions for the application of the LRCM to calculate the maximum value f_{max} and the approximate optimal x, x_{ap} for a function $f(x)$, are:

1) $f(x) = x \cdot g(x)$ in $\Re \times [0 \ \ x_{max}]$

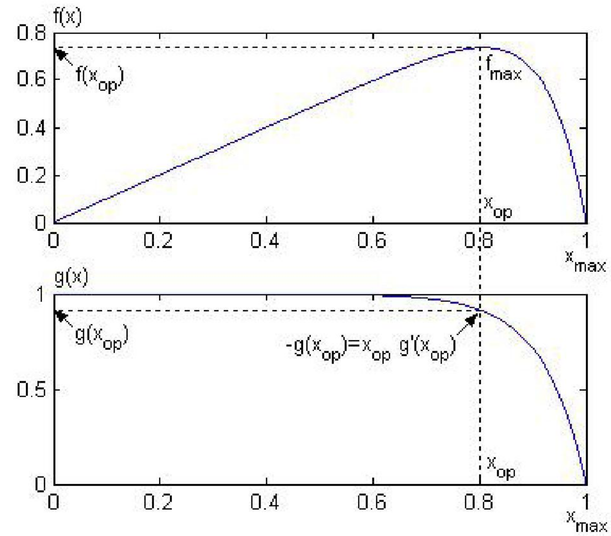

Fig. 1. Linear Reoriented Coordinates Method (LRCM).

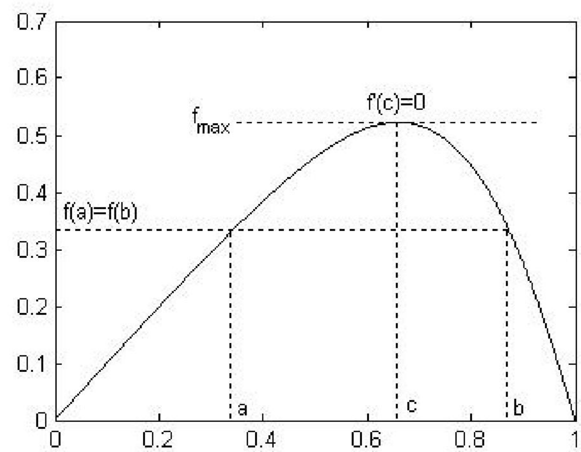

Fig. 2. Rolle's Theorem.

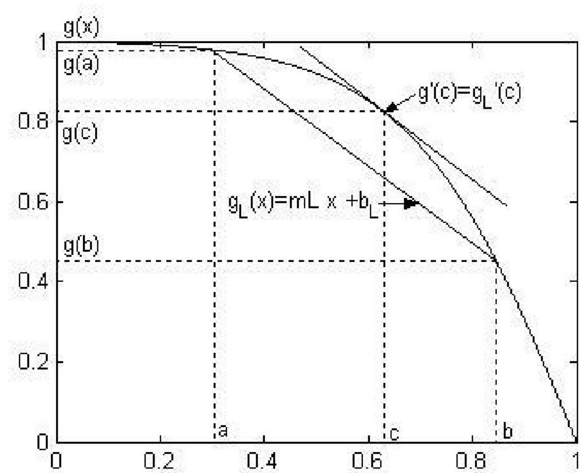

Fig. 3. Lagrange's Theorem.

2) $f \in C^1(\Re \times [0 \quad x_{max}])$
3) $g \in C^1(\Re \times [0 \quad x_{max}])$
4) $g'(x) < 0$ in $\Re \times [0 \quad x_{max}]$
5) $g''(x) \leq 0$ in $\Re \times [0 \quad x_{max}]$
6) *Corollary 1* is satisfied in $\{x \in \Re \times [0 \quad x_{max}]\}$
7) $g'(x_{op}) = -g(x_{op})/x_{op}$
8) $f_{max} = x_{op} \cdot g(x_{op})$

C. Approximation for x_{op} and f_{max}

Now, consider a function, $f(x)$, that satisfies the conditions for the LRCM hence it is desire to approximate x_{op}. The first step is to use the straight line given by (2) where $gl(x)$ is always positive in $\{x \in \Re \mid [0 \quad x_{max}]\}$. The derivative of $gl(x)$ with respect to x is always negative and unique in $\{x \in \Re \mid [0 \quad x_{max}]\}$. The derivatives of $gl(x)$ and $g(x)$ can be intersected in the point x_{ap} where it is the optimal point x_{op} plus an error, ϵ, as given in (3). For an small ϵ, the optimal value for x_{op} is approximated by (4), if ϵ is 0 then (4) is the solution for x_{op}.

$$gl(x) = bl + ml \cdot x = g(0) - \frac{g(0)}{x_{max}} \cdot x \tag{2}$$

$$gl'(x) = ml = -\frac{g(0)}{x_{max}} = gl'(x_{ap}) = gl'(x_{op} + \epsilon) \tag{3}$$

$$x_{op} \approx x_{op} + \epsilon = g'^{-1}\left(\frac{-g(0)}{x_{max}}\right) \tag{4}$$

The approximation of x_{op} is substituted in $f(x)$ to approximate f_{max} as given in (5). Finally, the error for the approximation of f_{max} is given by (6).

$$f(x_{ap}) = x_{ap} \cdot g(x_{ap}) = f_{ap} \approx f_{max} \tag{5}$$

$$Error = 100 \cdot \frac{f(x_{op}) - f(x_{ap})}{f(x_{op})} \tag{6}$$

D. Validation for the LRCM

Consider $f(x) = x \cdot g(x)$, and the derivative of $f(x)$ with respect to x, $f'(x) = g(x) + x \cdot g'(x)$ where $g(x)$ has the diffeomorphism property. Now using the Lagrange's Theorem and the Cauchy's Mean Value Theorem to find the optimal value x_{op} that it will produce the maximum value of $f(x) \Longrightarrow f_{max} = x_{op} \cdot g(x_{op}) = f(x_{op})$ in the domain $[0 \quad x_{max}]$ (Rolle's Thm.). Let's apply the Cauchy's Mean Value Theorem to $f(x)$ and $g(x)$ where both functions have the diffeomorphism property to solve for x_{op}.

$$f'(x_{op}) = \frac{f(r) - f(x_{max})}{r - x_{max}} = \frac{f(r)}{r - x_{max}} \tag{7}$$

$$g'(x_{op}) = \frac{g(r) - g(x_{max})}{r - x_{max}} = \frac{g(r)}{r - x_{max}} \tag{8}$$

$$r = \frac{f(r)}{g(r)} = \frac{f'(x_{op})}{g'(x_{op})} = \frac{g(x_{op})}{g'(x_{op})} + x_{op} \tag{9}$$

Using the *Corollary 2*, if $r = 0$ then the approximation for x_{op} is given by (10) and the approximation error is 0.

$$x_{op} = g'^{-1}\left(\frac{-g(0)}{x_{max}}\right) \tag{10}$$

Now, if $f(x)$ does not have the diffeomorphism property then x_{op} can not be solved (i.e. $x_{op} = f'^{-1}(0)$ is not possible to solve). Now, consider the function $g(x)$ to determine x_{op}, instead to use $f(x)$ because $f'(x) = g(x) + x \cdot g'(x)$. There is a linear slope (mL) with the same value as $g'(x_{op})$ to find f_{max}, $mL = g'(x_{op})$ (Lagrange's Thm.). Using Lagrange's Theorem, there is a function $gl(x) = ml \cdot x + bl$, where $gl(0) = g(0)$, $gl(x_{max}) = g(x_{max}) = 0$ and $gl'(x_{ap}) = g'(x_{ap})$, as given in (11) and (12).

$$mL = g'(x) \approx gl'(x) = \frac{-g(0)}{x_{max}} \tag{11}$$

$$x_{ap} \approx x_{op} \Longrightarrow x_{ap} = g'^{-1}\left(\frac{-g(0)}{x_{max}}\right) \tag{12}$$

Now, the approximate x_{op} can be calculated using (12)! Finally, an approximate f_{max} is calculated using x_{ap}, $f_{max} \approx f(x_{ap}) = x_{ap} \cdot g(x_{ap})$. The error of angle ε for x_{ap} and f_{max} will be calculated using (13),

$$\varepsilon = \tan^{-1}\left(g(x_{ap}) + x_{ap} \cdot g'(x_{ap})\right) \tag{13}$$

If $\varepsilon = 0$, then f_{max} is found, $g'(x_{op}) = gl(x_{op})$, $x_{ap} = x_{op}$ and the inverse map of the derivative of $f(x)$ is found.

IV. Additional Examples using the LRCM

Example 1: Consider the function $f(x)$ in $\{x \in \Re \mid [0 \quad r]\}$ given by (14) with the diffeomorphism property to find the maximum value f_{max} using differential calculus. The derivative of $f(x)$ is given by (15) hence the operation points x_{op} and f_{max} are given by (16).

$$f(x) = A \cdot x \cdot \left(r^2 - x^2\right)^{0.5} \tag{14}$$

$$f'(x) = A \cdot \left(r^2 - x^2\right)^{0.5} - A \cdot x^2 \cdot \left(r^2 - x^2\right)^{0.5} = 0 \tag{15}$$

$$\left(x_{op} = \frac{r}{\sqrt{2}}, \quad f(x_{op}) = \frac{A \cdot r^2}{2} = f_{max}\right) \tag{16}$$

Now let's find the maximum value for the same function $f(x)$ using LRCM.

1) Calculate $g'(x)$ using $g(x)$ where $g(r)$ is $A \cdot r$ and $g(0)$ is 0.

$$g(x) = A \cdot \left(r^2 - x^2\right)^{0.5} \tag{17}$$

$$g'(x) = -A \cdot x \cdot \left(r^2 - x^2\right)^{0.5} \tag{18}$$

2) Calculate $gl(x)$ using (2) then calculate $gl'(x)$

$$gl(x) = A \cdot r - A \cdot x \Longrightarrow gl'(x) = -A \tag{19}$$

3) Calculate x_{op} using the LRCM hence $g'(x) \approx gl'(x)$

$$x_{ap} = x_{op} = \frac{r}{\sqrt{2}} \tag{20}$$

4) To approximate f_{max}, x_{ap} is substituted in $f(x)$.

$$f(x_{ap}) = \frac{A \cdot r^2}{2} = f_{max} \tag{21}$$

5) Finally, ε is the final angle error for the approximation with $\varepsilon = 0°$ i.e. 0% of error for the approximation of x_{op}. Both results x_{op} and f_{max} can be solved and a symbolic solution is obtained with angle error of $0°$ i.e. $f'(x_{ap}) = 0$.

Example 2: A basic principle in microeconomics is to obtain the maximum profit and maximum revenues with the minimum costs. Consider the function (22) that describes the profit for the company **X** given the number of employees, n. The variable m is the maximum number of employees to be contracted that will not create a deficit to the company **X**, and k is a factor that relates the rate of profit per employee. It is desired to maximize the profits for a company only contracting the number of employees necessary to maximize the profit. Unfortunately, (22) does not have the diffeomorphism property. Now, if (22) is divided by n, (23) is obtained and has the diffeomorphism property that satisfies the conditions to apply the LRCM. The derivative of (23) given by (24) and the boundaries of (23) can be used to calculate the optimal number of employees, n_x to provide the maximum profit for the company **X**. Using the LRCM, n_x is calculated using (25). Now consider Fig. 4 and the LRCM where m is 52 and k is 10.06784 then n_x is 36 with a profit of $284,600\$$.

$$Profit(n) = n \cdot k - n \cdot (k-1) \cdot \left(\frac{k}{k-1}\right)^{\frac{n}{m}} \tag{22}$$

$$rate = k - (k-1) \cdot \left(\frac{k}{k-1}\right)^{\frac{n}{m}} \tag{23}$$

$$\frac{\partial rate}{\partial n} = \frac{1-k}{m} \cdot \ln\left(\frac{k}{k-1}\right) \cdot \left(\frac{k}{k-1}\right)^{\frac{n}{m}} \tag{24}$$

$$n_x = \frac{\ln(k-1) + \ln[\ln(k-1) - \ln(k)]}{\ln(k-1) - \ln(k)} \tag{25}$$

Example 3: The next example is to determine the inverse of a function $f(x)$ without diffeomorphism. The main goal is to determine the maximum rectangular area inside of the function $g(x)$. $g(x)$ describes the shape constraint relation for a floorplan for an 8-bit A/D converter and it is required to maximize the rectangular area inside of $g(x)$. Floorplan design is the first task in VLSI layout and perhaps the most important one [5]. In practical designs, the dimensions of some modules are restricted by physical designs and therefore can not be varied continuously [4]. $f(x)$ in $\{x \in \Re \mid [0 \;\; 25]\}$ represents the rectangular area occupied by a floorplan for an 8-bit A/D converter.

$$f(x) = \frac{x \cdot 25 \cdot \tan^{-1}(25-x)}{\tan^{-1}(25)} \tag{26}$$

Using differential calculus, it can be calculated $f'(x)$ and simplified to be solved by x but it cannot be solve for x, as given in (27).

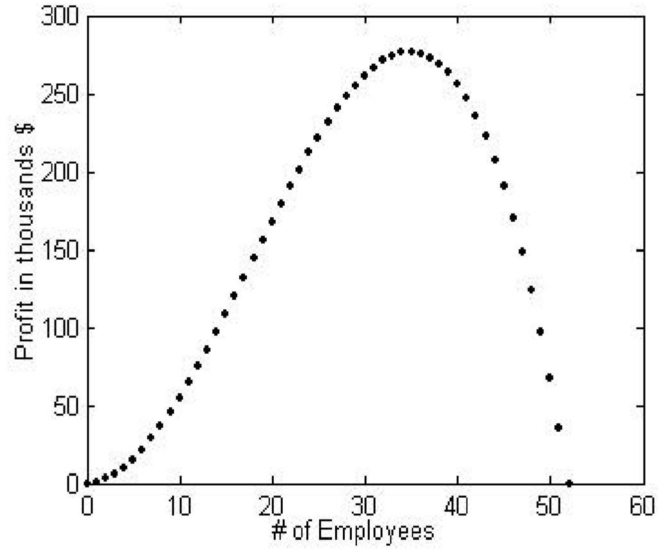

Fig. 4. Profit vs # of employees contracted by the company **X**.

$$\tan^{-1}(25-x) - \frac{x}{1+(25-x)^2} = 0 \tag{27}$$

Consider the LRCM using the following steps:

1) Calculate $g'(x)$ using $g(x)$, $g(25)$ is 0 and $g(0)$ is 25

$$g(x) = \frac{25 \cdot \tan^{-1}(25-x)}{\tan^{-1}(25)} \tag{28}$$

$$g'(x) = \frac{-25}{\tan^{-1}(25) + \tan^{-1}(25) \cdot (25-x)^2} \tag{29}$$

2) Calculate $gl(x)$ using (2) then calculate $gl'(x)$

$$gl(x) = 25 - x \Longrightarrow gl'(x) = -1 \tag{30}$$

3) Calculate the approximate value of x_{op} using the LRCM hence $g'(x) \approx gl'(x)$

$$x_{op} \approx x_{ap} = 25 - \sqrt{\frac{25}{\tan^{-1}(25)} - 1} \tag{31}$$

4) To approximate f_{max}, x_{ap} is substituted in $f(x)$ hence $f_{max} \approx f(x_{ap})$.

$$\begin{aligned} f_{max} \;\; \approx \;\; & \frac{25}{\tan^{-1}(25)} \cdot \tan^{-1}\left(\sqrt{\frac{25}{\tan^{-1}(25)} - 1}\right) \\ & \cdot \left(25 - \sqrt{\frac{25}{\tan^{-1}(25)} - 1}\right) \end{aligned} \tag{32}$$

5) The percentage of error for the approximation of f_{max} is less than 2.3% and was calculated using (6). This final result proved how good is the approximation for f_{max} considering that there is not analytical solution for (27).

Finally, the dimensions for the maximum rectangular area for a floorplan for an 8-bit A/D converter are for x-axis is

21.0845 units and for $g(x)$-axis is 21.5693 units.

Example 4: Consider Fig. 5 where it is shown the characteristic curve for a Fuel Cell where voltage output (V) versus the current density (A/cm^2) relationship are given and the area for the reactor is $1cm^2$. The voltage, V, and the power, P, in terms of the current, I, are described by (33) and (34). To obtain the maximum power, P_{max}, is required to solve the derivative of the power with respect to the current equal to zero.

$$V(I) = 0.3 + \frac{0.7}{\pi} \cdot \cos^{-1}\left(\frac{I}{0.7} - 1\right) \quad (33)$$

$$\begin{aligned} P(I) &= I \cdot V(I) = \\ & 0.3 \cdot I + \frac{0.7}{\pi} \cdot I \cdot \cos^{-1}\left(\frac{I}{0.7} - 1\right) \end{aligned} \quad (34)$$

$$\begin{aligned} \frac{\partial P(I)}{\partial I} &= 0.3 + \frac{0.7}{\pi} \cdot \cos^{-1}\left(\frac{I}{0.7} - 1\right) \\ & -\frac{I}{\pi} \cdot \left[1 - \left(\frac{I}{0.7} - 1\right)^2\right]^{-0.5} \end{aligned} \quad (35)$$

Unfortunately, it is not possible to solve (35) with respect to I due the absence of the diffeomorphism property. The LRCM can provide a good approximation for P_{max}.

$$\frac{\partial V(I)}{\partial I} = -\frac{1}{\pi} \cdot \left[1 - \left(\frac{I}{0.7} - 1\right)^2\right]^{-0.5} \quad (36)$$

$$Vl(I) = 1 - \frac{I}{2} \Longrightarrow \frac{\partial V(I)}{\partial I} = -\frac{1}{2} \quad (37)$$

After use (36) and (37), it is possible to solve for the approximate optimal current (I_{ap}) given by (38).

$$I_{ap} = 0.7 + 0.7 \cdot \sqrt{1 - \frac{4}{\pi^2}} \quad (38)$$

Finally, I_{ap} can be substituted in the voltage and power equations, (34) and (33). Table I shows the results of the LRCM for the voltage, current and power. The row with the approximation error values for each variable was calculated using (6).

TABLE I
COMPARISON FOR LRCM RESULTS AND OPTIMAL VALUES

	Voltage	Current	Power
Optimal	0.4902 V	1.1602 A	0.5687 W
Approx.	0.4538 V	1.2398 A	0.5626 W
Error	7.44 %	6.88 %	1.07 %

Example 5: Consider the function $g(x)$ described by (39). It is desire to calculate the maximum rectangular area inside of $g(x)$ $\forall$ x in $\{x \in \Re \mid [0 \ \ 4]\}$. The rectangular area inside of $g(x)$ can be calculated using $f(x) = x \cdot g(x)$, the derivative of $f(x)$ with respect to x is given by (40). Unfortunately is not possible to solve (40) equal to 0 but using the LRCM is possible to approximate the maximum rectangular area inside of $g(x)$.

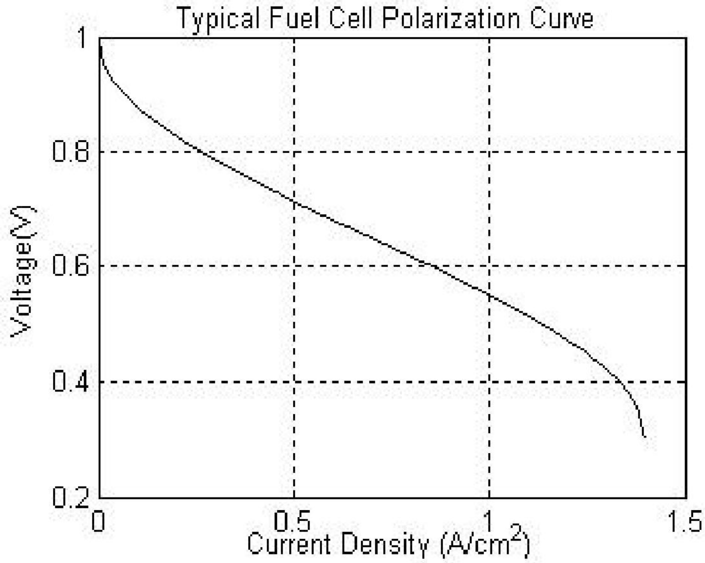

Fig. 5. V-I Characteristic Curve for a Fuel Cell.

$$g(x) = \exp(8) - \exp(4) + \exp(x) - \exp(2 \cdot x) \quad (39)$$

$$\begin{aligned} f'(x) &= \exp(8) - \exp(4) + (1 + x) \cdot \exp(x) \\ & -(1 + 2 \cdot x) \cdot \exp(2 \cdot x) \end{aligned} \quad (40)$$

1. Calculate the linear equation $gl(x)$ using the boundaries of $g(x)$ where $g(0)$ is $\exp(4) + \exp(8)$ and $g(4)$ is 0,

$$gl(x) = (\exp(4) + \exp(8)) \cdot \left(1 - \frac{x}{4}\right) \quad (41)$$

2. Determine $g'(x)$ and $gl'(x)$

$$g'(x) = exp(x) - 2 \cdot exp(2 \cdot x) \quad (42)$$

$$gl'(x) = -\frac{1}{4} \cdot (\exp(4) + \exp(8)) \quad (43)$$

3. Substitute $y = \exp(x)$ on $g'(x)$

$$g'(x) = exp(x) - 2 \cdot exp(2 \cdot x) = y - 2 \cdot y^2 \quad (44)$$

4. Using $g'(x)$ and $gl'(x)$ solve for y

$$\begin{aligned} g'(x_{ap}) &\approx gl'(x_{ap}) \Longrightarrow \\ & y - 2 \cdot y^2 \approx -\frac{1}{4} \cdot (\exp(8) + \exp(4)) \end{aligned} \quad (45)$$

$$y = \frac{1}{4} + \frac{1}{4} \cdot \sqrt{1 + 2 \cdot \exp(4) + 2 \cdot \exp(8)} = 19.7309 \quad (46)$$

5. Calculate x_{ap} then approximate the maximum area using x_{ap},

$$\begin{aligned} x_{ap} &= \ln(y) = \ln(19.7309) = 2.96411 \\ &\Longrightarrow f(x_{ap}) = x_{ap} \cdot g(x_{ap}) = 7,618.51 \end{aligned} \quad (47)$$

Finally, f_{max} is $7,631.62$ hence the percentage of error for the approximation $f(x_{ap})$ using (6) is 0.171524%. Again,

$f'(x) = 0$ is not possible to solve with respect to x due the absence of the diffeomorphism property in $f(x)$ but using the LRCM at least, it is possible to estimate the optimal value for x with small percentage of error!

Example 6: The next example is to determine the maximum power for a photovoltaic module given the I-V Characteristic Curve. This example was previously presented by [13]. Consider the function $P(V)$ for the application of photovoltaic modules given by (48) and (49). The photovoltaic module model has all the conditions for the application of the LRCM. $P(V)$ is the power delivered by a solar cell and $I(V)$ is the delivered current by a given voltage V given by the photovoltaic module where $\{V \in \Re \mid [0 \;\; 20]\}$.

$$P(V) = \frac{V - V \cdot \exp(0.5 \cdot V - 10)}{1 - \exp(-10)} \tag{48}$$

$$I(V) = \frac{1 - \exp(0.5 \cdot V - 10)}{1 - \exp(-10)} \tag{49}$$

The function $P(V)$ does not have the diffeomorphism property. The maximum power, P_{max}, is not possible to be calculated using (50) but using the LRCM at least can approximate P_{max} i.e. P_{max} is 13.884.

$$\frac{\partial P(V)}{\partial V} = \frac{1 - \exp(0.5 \cdot v - 10) - 0.5 \cdot V \cdot \exp(0.5 \cdot V - 10)}{1 - \exp(-10)} \tag{50}$$

1) Calculate $I'(V)$, using $I(V)$

$$\frac{\partial I(V)}{\partial V} = \frac{-0.5 \cdot V \cdot \exp(0.5 \cdot V - 10)}{1 - \exp(-10)} \tag{51}$$

2) Calculate $I_l(V)$ and $I_l'(V)$ where $I(0) = 1$ and $I(20) = 0$

$$I_l = 1 - \frac{V}{20} \Rightarrow I_l' = -\frac{1}{20} \tag{52}$$

3) Consider the condition for the LRCM, $I'(V) \approx I_l'(V)$

$$\begin{aligned} I'(V) &\approx I_l'(V) \Rightarrow -\exp(0.5 \cdot V - 10) \approx -0.05 \\ &\Rightarrow V_{ap} \approx 20 + 2 \cdot \ln(0.05) = 15.3940 \end{aligned} \tag{53}$$

4) To approximate P_{max}, V_{ap} is substituted in (48) hence $P_{map} = I(V_{ap}) \cdot V_{ap} = 13.856W$.

5) Finally, the error for the approximation of P_{max}, is less than 0.2%.

The final result shows that the LRCM is a very good method for the approximation of the maximum power, P_{max}, produced by a photovoltaic module considering that the error for the approximation is less than 0.2%.

V. Conclusion

This paper presented a method called Linear Reoriented Coordinates Method (LRCM). The LRCM is a nontraditional method to be applied for functions without the diffeomorphism property. With the use of the LRCM, solutions to obtain the approximate maximum value f_{max} for a function, $f(x) = x \cdot g(x)$, will be obtained using $g(x)$ and the linear equation, $gl(x)$. Another advantage is that the LRCM can be used to calculate the symbolic inverse for the one dimensional map f'^{-1}. The LRCM can provide analytical and symbolic solutions very close to the Lagrange Inverse Theorem or differential calculus methods without the use of Taylor series, continuous fractions or other type of approximations. Additionally, the LRCM can be integrated to other optimization methods. Finally, the LRCM is more practical for simulations due to the symbolic solutions. This method may be applied to other fields like math, geology, civil engineering, economy and mechanical engineering, etc.

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